

Larmor times for transmission and reflection

N. L. Chuprikov

*Tomsk State Pedagogical University, 634041, Tomsk, Russia**

(Dated: February 1, 2008)

On the basis of quant-ph/0405028 we define the Larmor times for transmission and reflection. These times are valid both for the stationary and time-dependent scattering processes, without any restrictions on the shape of Gaussian-like wave packets. We show that in the stationary case both the Larmor times coincide with the corresponding dwell times obtained in quant-ph/0502073. The Larmor-time concept gives the way to verify the approach presented in quant-ph/0405028.

PACS numbers: 03.65.Ca, 03.65.Xp

I. INTRODUCTION

By our approach (see [1]) the state of the quantum ensemble of identically prepared particles tunneling through a symmetrical time-independent potential barrier represents a coherent superposition of two states: that of particles which are finally transmitted through the barrier, and that of particles which are finally reflected by it. This is valid for any stage of this scattering process. For a given symmetrical potential barrier and initial state of the whole ensemble of particles there is an unique pair of solutions to the Schrödinger equation, which describe the coherently evolved alternative sub-processes, transmission and reflection.

Note, the fact that the above decomposition is valid, in particular, for the first stage of scattering does not at all mean that the future of a starting particle is predetermined in our approach. Of course, this is not the case. The knowledge of the wave functions for transmission and reflection does not permit us to predict the behavior of one particle in one experiment. As regards the subensembles of transmitted and reflected particles, their history has a deterministic character and, thereby, is predictable. This agrees entirely with the basic principles of quantum mechanics.

Our approach opens new possibilities in solving the so-called tunneling time problem to have been of great controversy for the last decades. In [1] we have introduced the so-called (exact and asymptotic) group tunneling times for transmission and reflection, and in [2] we have defined the dwell times for these two sub-processes. These time concepts supplement each other. The group tunneling times for transmission and reflection are formally valid in the general case. However, in fact they more fit only for timing a particle in a well-localized state. As regards sufficiently wide (in x -space) wave packets, the corresponding expectation value of the particle's position is badly defined. Moreover, in this case, reflected particles do not enter, on the average, into the barrier region. Thus, the exact group reflection time is zero in

this case, what is evident to be a very rough description of the reflection process.

In the case of infinitely wide wave packets, the time spent by a particle in the barrier region is described by the dwell time. However, this time concept, too, has shortcomings: 1) it has been introduced intuitively, rather than from the first principles; 2) and, besides, it is valid only for the stationary scattering problem.

In addition to the above, there is once more shortcoming to be common both for the group and dwell time concepts: they do not give the way of measuring the above tunneling times for transmission and reflection. The first concept does not give the way of measuring the (average) time of entering to-be-transmitted (or to-be-reflected) particles in the barrier region. Similarly, the second one does not suggest the way of how experimentally to extract the information about the individual properties of the subensembles of to-be-transmitted or to-be-reflected particles in the barrier region.

It is evident that to distinguish to-be-transmitted and to-be-reflected particles is impossible. In order to measure the (average) times spent by the subensembles of transmitted and reflected particles in the barrier region, we have to place measuring detectors far from the barrier region. Perhaps, the most suitable physical effect which can be assumed as a basis for such measurements is the Larmor precession of the particle's spin, in a weak magnetic field confined to the barrier region.

As is known, the idea to use the Larmor precession as clocks was proposed by Baz' [3] and developed later by Rybachenko [4] and Büttiker [5] (see also [6, 7]). Our aim is to define the Larmor times for transmission and reflection, on the basis of the wave functions found in [1]. Besides, we consider, of great importance for understanding the tunneling phenomenon is to answer the questions posed in [6]. One of them concerns the appearance of a superluminal transmission of a particle through sufficiently opaque and wide potential barriers (the Hartman effect). Another question relates to the influence of the magnetic field confined to a spatial region behind the barrier, on the spin of reflected particles.

Apart from these two questions one has to explain the effect found by Büttiker (see [5]). He showed that the magnetic field aligns the particle's spin with the field. This result is evident to be paradoxical too. For the oper-

*Also at Physics Department, Tomsk State University.

ator of the spin z -component commutes with the Hamiltonian in this problem. The expectation value of this operator should be constant individually both for transmitted and reflected particles.

We have to stress once more (see [1]) that averaging any observable (and the well-known Born's formula for the expectation values) has a physical sense only for the so-called "elementary" states, which cannot be decomposed onto macroscopically distinguishable ones. The notion of the integrals of motion is valid, too, only for "elementary" states. In the case considered the states of the subensembles of transmitted and reflected particles are just the states of such kind. As regards averaging over the combined state of the whole ensemble of tunneling particles, this procedure is meaningless for it. Just calculating the "expectation values" of physical quantities, for "combined" states, leads to paradoxes.

II. TUNNELING A SPINNING PARTICLE THROUGH A SYMMETRICAL FINITE POTENTIAL BARRIER: SETTING THE PROBLEM

Let us consider the quantum ensemble of electrons moving along the x -axis and interacting with the symmetrical time-independent potential barrier $V(x)$ and small magnetic field (parallel to the z -axis) confined to the finite spatial interval $[a, b]$ ($a > 0$); $V(x - x_c) = V(x_c - x)$; $x_c = (b + a)/2$; $d = b - a$ is the barrier width. Let this ensemble be a mixture of two parts. One of them consists from electrons with spin parallel to the magnetic field. Another is formed from particles with spin down.

Let at $t = 0$ the state of this mixture is described by the spinor

$$\Psi_{full}^{(0)}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \psi_{full}^{(0)}(x), \quad (1)$$

where $\psi_{full}^{(0)}(x)$ is a normalized function. Without loss of generality we will suppose that

$$\langle \psi_{full}^{(0)} | \hat{x} | \psi_{full}^{(0)} \rangle = 0, \quad \langle \psi_{full}^{(0)} | \hat{p} | \psi_{full}^{(0)} \rangle = \hbar k_0 > 0,$$

$$\langle \psi_{full}^{(0)} | \hat{x}^2 | \psi_{full}^{(0)} \rangle = l_0^2,$$

here l_0 is the wave-packet's half-width at $t = 0$ ($l_0 \ll a$); \hat{x} and \hat{p} are the operators of the particle's position and momentum, respectively.

So, we will consider the case, when the initial spin coherent state (1) is the engine state of σ_x with the eigenvalue 1 (the average spin of the ensemble of incident particles is oriented along the x -direction); hereinafter, σ_x , σ_y and σ_z are the Pauli spin matrices.

For electrons with spin up (down), the potential barrier effectively decreases (increases), in height, by the value

$\hbar\omega_L/2$; here ω_L is the frequency of the Larmor precession; $\omega_L = 2\mu B/\hbar$, μ denotes the magnetic moment. The corresponding Hamiltonian has the following form,

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(x) - \frac{\hbar\omega_L}{2}\sigma_z, \quad \text{if } x \in [a, b];$$

$$\hat{H} = \frac{\hat{p}^2}{2m}, \quad \text{otherwise}; \quad (2)$$

here m is the mass of a particle. For $t > 0$, due to the influence of the magnetic field, the states of particles with spin up and down become different. The probability to pass the barrier is different for them. Let for any value of t the spinor to describe the state of particles read as

$$\Psi_{full}(x, t) = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_{full}^{(\uparrow)}(x, t) \\ \psi_{full}^{(\downarrow)}(x, t) \end{pmatrix}. \quad (3)$$

In accordance with [1], each of these two spinor components can be uniquely presented as a coherent superposition of two probability fields to describe transmission and reflection:

$$\psi_{full}^{(\uparrow)}(x, t) = \psi_{tr}^{(\uparrow)}(x, t) + \psi_{ref}^{(\uparrow)}(x, t);$$

$$\psi_{full}^{(\downarrow)}(x, t) = \psi_{tr}^{(\downarrow)}(x, t) + \psi_{ref}^{(\downarrow)}(x, t). \quad (4)$$

(remind (see [1]) that $\psi_{ref}(x, t) \equiv 0$ for $x \geq x_c$). As a consequence, the same decomposition takes place for spinor (3):

$$\Psi_{full}(x, t) = \Psi_{tr}(x, t) + \Psi_{ref}(x, t). \quad (5)$$

We will suppose that all the wave functions for transmission and reflection are known. It is important to stress here (see [1]) that

$$\langle \psi_{full}^{(\uparrow\downarrow)}(x, t) | \psi_{full}^{(\uparrow\downarrow)}(x, t) \rangle = T^{(\uparrow\downarrow)} + R^{(\uparrow\downarrow)} = 1 \quad (6)$$

where

$$T^{(\uparrow\downarrow)} = \langle \psi_{tr}^{(\uparrow\downarrow)}(x, t) | \psi_{tr}^{(\uparrow\downarrow)}(x, t) \rangle = \text{const}; \quad (7)$$

$$R^{(\uparrow\downarrow)} = \langle \psi_{ref}^{(\uparrow\downarrow)}(x, t) | \psi_{ref}^{(\uparrow\downarrow)}(x, t) \rangle = \text{const}; \quad (8)$$

$T^{(\uparrow\downarrow)}$ and $R^{(\uparrow\downarrow)}$ are the (real) transmission and reflection coefficients, respectively, for particles with spin up (\uparrow) and down (\downarrow). Let further $T = (T^{(\uparrow)} + T^{(\downarrow)})/2$ and $R = (R^{(\uparrow)} + R^{(\downarrow)})/2$ be quantities to describe the whole ensemble of particles.

III. TIME EVOLUTION OF THE SPIN POLARIZATION OF TUNNELING PARTICLES

To study the time evolution of the average particle's spin, we have to find the expectation values of the spin projections \hat{S}_x , \hat{S}_y and \hat{S}_z . Note, for any t

$$\langle \hat{S}_x \rangle_{full} \equiv \frac{\hbar}{2} \sin(\theta_{full}) \cos(\phi_{full})$$

$$= \hbar \cdot \Re(\langle \psi_{full}^{(\uparrow)} | \psi_{full}^{(\downarrow)} \rangle); \quad (9)$$

$$\begin{aligned} \langle \hat{S}_y \rangle_{full} &\equiv \frac{\hbar}{2} \sin(\theta_{full}) \sin(\phi_{full}) \\ &= \hbar \cdot \Im(\langle \psi_{full}^{(\uparrow)} | \psi_{full}^{(\downarrow)} \rangle); \end{aligned} \quad (10)$$

$$\begin{aligned} \langle \hat{S}_z \rangle_{full} &\equiv \frac{\hbar}{2} \cos(\theta_{full}) \\ &= \frac{\hbar}{2} [\langle \psi_{full}^{(\uparrow)} | \psi_{full}^{(\uparrow)} \rangle - \langle \psi_{full}^{(\downarrow)} | \psi_{full}^{(\downarrow)} \rangle]. \end{aligned} \quad (11)$$

Similar expressions are valid for transmission and reflection:

$$\langle \hat{S}_x \rangle_{tr} = \frac{\hbar}{T} \Re(\langle \psi_{tr}^{(\uparrow)} | \psi_{tr}^{(\downarrow)} \rangle); \quad (12)$$

$$\langle \hat{S}_y \rangle_{tr} = \frac{\hbar}{T} \Im(\langle \psi_{tr}^{(\uparrow)} | \psi_{tr}^{(\downarrow)} \rangle); \quad (13)$$

$$\langle \hat{S}_z \rangle_{tr} = \frac{\hbar}{2T} (\langle \psi_{tr}^{(\uparrow)} | \psi_{tr}^{(\uparrow)} \rangle - \langle \psi_{tr}^{(\downarrow)} | \psi_{tr}^{(\downarrow)} \rangle); \quad (14)$$

$$\langle \hat{S}_x \rangle_{ref} = \frac{\hbar}{R} \Re(\langle \psi_{ref}^{(\uparrow)} | \psi_{ref}^{(\downarrow)} \rangle); \quad (15)$$

$$\langle \hat{S}_y \rangle_{ref} = \frac{\hbar}{R} \Im(\langle \psi_{ref}^{(\uparrow)} | \psi_{ref}^{(\downarrow)} \rangle); \quad (16)$$

$$\langle \hat{S}_z \rangle_{ref} = \frac{\hbar}{2R} (\langle \psi_{ref}^{(\uparrow)} | \psi_{ref}^{(\uparrow)} \rangle - \langle \psi_{ref}^{(\downarrow)} | \psi_{ref}^{(\downarrow)} \rangle); \quad (17)$$

Note, at $t = 0$, for spinor (1), $\theta_{full} = \pi/2$, $\phi_{full} = 0$. However, this is not the case for transmission and reflection. Namely, at $t = 0$ we have

$$\phi_{tr}^{(0)} = \arctan \left(\frac{\Im(\langle \psi_{tr}^{(\uparrow)}(x, 0) | \psi_{tr}^{(\downarrow)}(x, 0) \rangle)}{\Re(\langle \psi_{tr}^{(\uparrow)}(x, 0) | \psi_{tr}^{(\downarrow)}(x, 0) \rangle)} \right); \quad (18)$$

$$\begin{aligned} \theta_{tr}^{(0)} &= \arccos \left(\langle \psi_{tr}^{(\uparrow)}(x, 0) | \psi_{tr}^{(\uparrow)}(x, 0) \rangle \right. \\ &\quad \left. - \langle \psi_{tr}^{(\downarrow)}(x, 0) | \psi_{tr}^{(\downarrow)}(x, 0) \rangle \right); \end{aligned} \quad (19)$$

$$\phi_{ref}^{(0)} = \arctan \left(\frac{\Im(\langle \psi_{ref}^{(\uparrow)}(x, 0) | \psi_{ref}^{(\downarrow)}(x, 0) \rangle)}{\Re(\langle \psi_{ref}^{(\uparrow)}(x, 0) | \psi_{ref}^{(\downarrow)}(x, 0) \rangle)} \right); \quad (20)$$

$$\begin{aligned} \theta_{ref}^{(0)} &= \arccos \left(\langle \psi_{ref}^{(\uparrow)}(x, 0) | \psi_{ref}^{(\uparrow)}(x, 0) \rangle \right. \\ &\quad \left. - \langle \psi_{ref}^{(\downarrow)}(x, 0) | \psi_{ref}^{(\downarrow)}(x, 0) \rangle \right). \end{aligned} \quad (21)$$

Since the norms of $\psi_{tr}^{(\uparrow)}(x, t)$, $\psi_{tr}^{(\downarrow)}(x, t)$, $\psi_{ref}^{(\uparrow)}(x, t)$ and $\psi_{ref}^{(\downarrow)}(x, t)$ are constant, $\theta_{tr}(t) = \theta_{tr}^{(0)}$ and $\theta_{ref}(t) = \theta_{ref}^{(0)}$

for any value of t . For the (constant) z -components of spin we have

$$\begin{aligned} \langle \hat{S}_z \rangle_{tr}(t) &= \hbar \frac{T^{(\uparrow)} - T^{(\downarrow)}}{2T}; \\ \langle \hat{S}_z \rangle_{ref}(t) &= \hbar \frac{R^{(\uparrow)} - R^{(\downarrow)}}{2R}. \end{aligned} \quad (22)$$

By our approach, the assertion made in [5, 7] that the magnetic field aligns the particle's spin with the field is incorrect. We see that the influence of the magnetic field on spin is twofold. Firstly, it causes the Larmor precession. Secondly, it disturbs the balance between transmitted and reflected particles with the up and down spins. As a result, transmitted and reflected particles possess a nonzero z -component of the average spin, though for the case considered it is zero for the whole ensemble of particles. However, we have to stress that the **ensemble** of transmitted (reflected) particles has the constant value of $\langle \hat{S}_z \rangle_{tr}(t)$ ($\langle \hat{S}_z \rangle_{ref}(t)$) in the course of its (deterministic) motion. This spin component must be constant, because the operator \hat{S}_z commutes with Hamiltonian (2) (we have here to bear in mind that the individual probability fields for transmission and reflection, though superposed, evolve independently each other).

To answer the question posed in [6], we have to pay reader's attention on the following. As is seen from the above, of no importance is where the magnetic field is localized: switching on the infinitesimal magnetic field, in any spatial region, must change simultaneously the average spin of transmitted and reflected particles.

IV. LARMOR PRECESSION UNDER THE INFINITESIMAL MAGNETIC FIELD CONFINED TO THE BARRIER REGION

So, the only effect caused by the magnetic field in this scattering process is the Larmor precession of the average particle's spin. And our next step is to use these "clocks" in order to define the time spent by particles in the barrier region. In doing so, we have to take into account that the initial position of the clock-pointer is not zero.

Following [5, 7] we will suppose that the applied magnetic field is infinitesimal. Our first step is to find the derivations $d\phi_{tr}/dt$ and $d\phi_{ref}/dt$. For this purpose we will use the Ehrenfest equations for the average spin of particles. One can show that

$$\begin{aligned} \frac{d \langle \hat{S}_x \rangle_{tr}}{dt} &= -\hbar \omega_L \int_a^b \Im[(\psi_{tr}^{(\uparrow)}(x, t))^* \psi_{tr}^{(\downarrow)}(x, t)] dx \\ \frac{d \langle \hat{S}_y \rangle_{tr}}{dt} &= \hbar \omega_L \int_a^b \Re[(\psi_{tr}^{(\uparrow)}(x, t))^* \psi_{tr}^{(\downarrow)}(x, t)] dx \end{aligned} \quad (23)$$

$$\frac{d \langle \hat{S}_x \rangle_{ref}}{dt} = -\hbar \omega_L \int_a^{x_c} \Im[(\psi_{ref}^{(\uparrow)}(x, t))^* \psi_{ref}^{(\downarrow)}(x, t)] dx$$

$$\frac{d \langle \hat{S}_y \rangle_{ref}}{dt} = \hbar \omega_L \int_a^{x_c} \Re[(\psi_{ref}^{(\uparrow)}(x, t))^* \psi_{ref}^{(\downarrow)}(x, t)] dx.$$

Note,

$$\phi_{tr} = \arctan \left(\frac{\langle \hat{S}_y \rangle_{tr}}{\langle \hat{S}_x \rangle_{tr}} \right), \phi_{ref} = \arctan \left(\frac{\langle \hat{S}_y \rangle_{ref}}{\langle \hat{S}_x \rangle_{ref}} \right)$$

Hence

$$\frac{d\phi_{tr}}{dt} = \frac{\frac{d\langle \hat{S}_y \rangle_{tr}}{dt} \langle \hat{S}_x \rangle_{tr} - \langle \hat{S}_y \rangle_{tr} \frac{d\langle \hat{S}_x \rangle_{tr}}{dt}}{\langle \hat{S}_x \rangle_{tr}^2 + \langle \hat{S}_y \rangle_{tr}^2}.$$

The same expression takes place for reflection.

However, in the case of the initial condition chosen and infinitesimal magnetic field, when

$$\frac{|\langle \hat{S}_y \rangle_{tr}|}{|\langle \hat{S}_x \rangle_{tr}|} \ll 1 \quad \text{and} \quad \frac{|\langle \hat{S}_y \rangle_{ref}|}{|\langle \hat{S}_x \rangle_{ref}|} \ll 1,$$

these expressions are essentially simplified —

$$\frac{d\phi_{tr}}{dt} = \frac{1}{\langle \hat{S}_x \rangle_{tr}} \cdot \frac{d\langle \hat{S}_y \rangle_{tr}}{dt};$$

$$\frac{d\phi_{ref}}{dt} = \frac{1}{\langle \hat{S}_x \rangle_{ref}} \cdot \frac{d\langle \hat{S}_y \rangle_{ref}}{dt}.$$

Considering Exps. (12)-(17) and (23), we obtain

$$\frac{d\phi_{tr}}{dt} = \omega_L \frac{\int_a^b \Re[(\psi_{tr}^{(\uparrow)}(x, t))^* \psi_{tr}^{(\downarrow)}(x, t)] dx}{\int_{-\infty}^{\infty} \Re[(\psi_{tr}^{(\uparrow)}(x, t))^* \psi_{tr}^{(\downarrow)}(x, t)] dx};$$

$$\frac{d\phi_{ref}}{dt} = \omega_L \frac{\int_a^{x_c} \Re[(\psi_{ref}^{(\uparrow)}(x, t))^* \psi_{ref}^{(\downarrow)}(x, t)] dx}{\int_{-\infty}^{x_c} \Re[(\psi_{ref}^{(\uparrow)}(x, t))^* \psi_{ref}^{(\downarrow)}(x, t)] dx}.$$

Or, taking into account that in the first order approximation, for the infinitesimal magnetic field, when $\psi_{tr}^{(\uparrow)}(x, t) = \psi_{tr}^{(\downarrow)}(x, t) = \psi_{tr}(x, t)$ and $\psi_{ref}^{(\uparrow)}(x, t) = \psi_{ref}^{(\downarrow)}(x, t) = \psi_{ref}(x, t)$, we have

$$\frac{d\phi_{tr}}{dt} \approx \omega_L \frac{\int_a^b |\psi_{tr}(x, t)|^2 dx}{\int_{-\infty}^{\infty} |\psi_{tr}(x, t)|^2 dx} > 0;$$

$$\frac{d\phi_{ref}}{dt} \approx \omega_L \frac{\int_a^{x_c} |\psi_{ref}(x, t)|^2 dx}{\int_{-\infty}^{x_c} |\psi_{ref}(x, t)|^2 dx} > 0.$$

As is seen, the clocks chosen operate properly: their clock-pointers rotate in one direction.

As is supposed in our setting the problem, both at the initial and final moments of time the ensemble of particles does not interact with the potential barrier and magnetic field. In this case, without loss of exactness, the

angles of rotation ($\Delta\phi_{tr}$ and $\Delta\phi_{ref}$) of spin under the magnetic field, in the course of a completed scattering, can be written in the form,

$$\Delta\phi_{tr} = \frac{\omega_L}{T} \int_{-\infty}^{\infty} dt \int_a^b dx |\psi_{tr}(x, t)|^2 \quad (24)$$

$$\Delta\phi_{ref} = \frac{\omega_L}{R} \int_{-\infty}^{\infty} dt \int_a^{x_c} dx |\psi_{ref}(x, t)|^2. \quad (25)$$

On the other hand, we have to bear in mind that the times spent, on the average, by transmitted and reflected particles in the barrier region are finite. Let τ_{tr}^L and τ_{ref}^L be the corresponding times for transmission and reflection. Thus, $\Delta\phi_{tr} = \omega_L \tau_{tr}^L$ and $\Delta\phi_{ref} = \omega_L \tau_{ref}^L$. Comparing these expressions with (24) and (25), respectively, we eventually obtain

$$\tau_{tr}^L = \frac{1}{T} \int_{-\infty}^{\infty} dt \int_a^b dx |\psi_{tr}(x, t)|^2 \quad (26)$$

$$\tau_{ref}^L = \frac{1}{R} \int_{-\infty}^{\infty} dt \int_a^{x_c} dx |\psi_{ref}(x, t)|^2. \quad (27)$$

These are just the searched-for definitions of the Larmor times for transmission and reflection.

Unlike the group and dwell time concepts, the Larmor-time concept gives the basis to verify our formalism. Indeed, let for the above setting the problem we have measured the spin of transmitted particles. Let the azimuthal angle $\phi_{tr}(t)$ at $t \rightarrow \infty$ be $\phi_{tr}^{(\infty)}$. By our approach, the final angle should be equal to $\phi_{tr}^{(\infty)} = \phi_{tr}^{(0)} + \Delta\phi_{tr}$ (see Exps. (18) and (24)).

As is seen, there is the problem to distinguish the inputs $\phi_{tr}^{(0)}$ and $\Delta\phi_{tr}$. However, one can show, for example, that in the case of tunneling a particle through an opaque rectangular potential barrier (just where the so-called Hartman effect "appears") $|\phi_{tr}^{(0)}| \ll \Delta\phi_{ref} \ll \Delta\phi_{tr}$. Thus, this case is most suitable and interesting for checking the above expressions for the Larmor times.

V. CONNECTION BETWEEN THE LARMOR AND DWELL TIMES

Note firstly that Exps. (26) and (27) for the Larmor times coincide by form with those obtained in the known approaches (see, e.g., [6, 8, 9, 10, 11, 12]) as the dwell times. However, there are deep differences between the known dwell-time concepts and Larmor times presented here. Namely, 1) our Larmor times distinguish between transmitted and reflected particles; 2) they are exact, rather than asymptotic values; 3) they were derived for any incident Gaussian-like wave packets, which may contain harmonics with the zero momentum; 4) unlike the known times for transmission and reflection, they are real and non-negative.

Let us now display the connection between Larmor time (26), (27) and the dwell times derived in [2]. Let us consider the case of transmission. For this purpose let us write down the wave function $\psi_{tr}(x, t)$ in the form

$$\psi_{tr}(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) \psi_{tr}(x, k) e^{-iE(k)t/\hbar} dk, \quad (28)$$

where $E(k) = \hbar^2 k^2 / 2m$; $\psi_{tr}(x, k)$ is the stationary wave function for transmission (see [1]); $A(k)$ is a real Gaussian-like function.

Let us transform the integral in (26),

$$I = \int_{-\infty}^{\infty} dt \int_a^b dx |\psi_{tr}(x, t)|^2.$$

Considering Exp. (28) and integrating on t , we obtain

$$I = \frac{\hbar}{\pi} \int_{-\infty}^{\infty} dk' dk A(k') A(k) \int_a^b dx \psi_{tr}^*(x, k') \psi_{tr}(x, k) \\ \times \lim_{\Delta t \rightarrow \infty} \frac{\sin[(E(k') - E(k))\Delta t/\hbar]}{E(k') - E(k)}.$$

However,

$$\lim_{\Delta t \rightarrow \infty} \frac{\sin[(E(k') - E(k))\Delta t/\hbar]}{E(k') - E(k)} = \frac{\pi}{\hbar} \delta[(E(k') - E(k))/\hbar] \\ = \frac{\pi m}{\hbar^2 k} [\delta(k' - k) - \delta(k' + k)].$$

Hence

$$I = \frac{m}{\hbar} \int_{-\infty}^{\infty} dk \frac{A(k)}{k} \int_a^b dx \left[A(k) \psi_{tr}^*(x, k) - A(-k) \psi_{tr}^*(x, -k) \right] \psi_{tr}(x, k). \quad (29)$$

The integrand in this expression is evident to be non-singular at $k = 0$.

Let us consider the stationary case: $A(k) = \delta(k - k_0)$ (note, $k_0 > 0$). Then

$$I = \frac{m}{\hbar k_0} \int_a^b dx |\psi_{tr}(x, k_0)|^2.$$

And, lastly, substituting this expression in (26), we obtain

$$\tau_{tr}^L = \frac{m}{\hbar k_0 T} \int_a^b dx |\psi_{tr}(x, k_0)|^2.$$

However, this is just the dwell time for transmission ([1]).

In the same way one can show that in the stationary case the Larmor and dwell times for reflection are equal too:

$$\tau_{ref}^L = \frac{m}{\hbar k_0 R} \int_a^{x_c} dx |\psi_{ref}(x, k_0)|^2.$$

VI. CONCLUSION

We have introduced the Larmor times to be valid for any symmetrical finite potential barriers and Gaussian-like wave packets. The Larmor-time concept presented differs from the known analogs. We show that in the case of the stationary tunneling problem the Larmor times coincide with the dwell times obtained in our previous paper. The formalism presented can be, in principle, verified experimentally.

For example, it is very important to measure the Larmor times for particles tunneling through an opaque rectangular barrier. By our approach, in this case the Larmor time for transmission should be very large, in comparison with the predictions of other approaches.

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- [1] N. L. Chuprikov, Wave functions and characteristic times for transmission and reflection, quant-ph/0405028.
 - [2] N. L. Chuprikov, Dwell times for transmission and reflection, quant-ph/0502073.
 - [3] A. I. Baz', Sov. J. Nucl. **4**, 182; **5**, 161 (1967).
 - [4] V. F. Rybachenko, Sov. J. Nucl. **5**, 635 (1967).
 - [5] M. Buttiker, Phys. Rev. B **27** 6178 (1983).
 - [6] C. R. Leavens and G. C. Aers, Phys. Rev. B **40** 5387 (1989).
 - [7] Zhi-Jian Li, J. Q. Liang, and D. H. Kobe, Phys. Rev. A **64** 043112 (2001).
 - [8] C.A.A. de Carvalho, H.M. Nussenzveig, Physics Reports, **364** 83-174 (2002).
 - [9] E. H. Hauge, J. P. Falck, and T. A. Fjeldly, Phys. Rev. B **36**, 4203 (1987).
 - [10] C. R. Leavens and G. C. Aers, Phys. Rev. B **39**, 1202 (1989).
 - [11] A.M. Steinberg, Phys. Rev. A **52**, 32 (1995); Phys. Rev. Lett. **74**, 2405 (1995).
 - [12] Yakir Aharonov, Noam Erez, and Benni Reznik, Phys. Rev. A **65**, 052124 (2002).